Zentralblatt–MATH

Allenby, R.B.J.T.; Tang, C.Y.

On the residual finiteness of certain polygonal products. (English)

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A polygon of groups in an amalgam of groups $\mathcal{G}_1, \mathcal{G}_2, ..., \mathcal{G}_n, \mathcal{G}_{n+1} = \mathcal{G}_1$, say, in which only $\mathcal{G}_i$ and $\mathcal{G}_{i+1}$, for $i = 1, 2, ..., n$, are allowed to have nontrivial intersections, say $\mathcal{G}_i \cap \mathcal{G}_{i+1} = \mathcal{H}_i$ (this is not the authors' notation). The constituent groups $\mathcal{G}_i$ are called vertex groups, the intersections $\mathcal{H}_i$ edge groups. The canonic group (or generalized free product) of the amalgam is called the polygonal product. It is known that when $n = 3$, (the case of "triangle products"), the amalgam may fail to be embedded in the product, while for $n \geq 4$ the polygonal product embeds the polygon. For the sake of simplicity the authors concentrate on the case that $n = 4$ (a "square product"); and they further restrict consideration to the case where consecutive edge groups have trivial intersection, and all edge groups are cyclic. They show by a simple example (Example 4.1, with the vertex groups all dihedral of order 8 and the edge groups of order 2) that the square product of nilpotent groups need not be residually nilpotent. However, if the vertex groups are finite p-groups and at least one pair of opposite edge groups consists of groups of order p, then the square product is shown to be residually finite (Theorem 4.4). Also if the vertex groups are abelian, the square product (still with disjoint cyclic edge groups) is residually finite (Theorem 3.4).

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