Zentralblatt–MATH

Allman, Elizabeth S.; Hamilton, Emily

Abelian subgroups of finitely generated Kleinian groups are separable. (English)

Bull. Lond. Math. Soc. 31, No.2, 163-172 (1999). [ISSN 0024-6093]
http://www.journals.cup.org/owa_dba/owa/volumes?sjid=BLM

A subgroup H of G is called separable in G if it is an intersection of finite-index subgroups of G. This generalizes the well-known property of residual finiteness, which is the condition that the trivial subgroup is separable in G. If every subgroup of G is separable in G, then G is called locally extended residually finite, or LERF. Examples are known of fundamental groups of compact 3-manifolds that are not LERF, but it is unknown whether fundamental groups of closed hyperbolic 3-manifolds are LERF. The main result of the paper is that if Γ is the fundamental group of a hyperbolic 3-orbifold, then abelian subgroups of Γ are separable in Γ. This was already known for maximal abelian subgroups.

The proof combines algebraic and geometric methods. Using various results from 3-dimensional topology and Kleinian group theory, the authors reduce to the case when Γ is torsion-free and geometrically finite, the trace field of Γ is a number field, and every parabolic element of Γ is contained in a subgroup isomorphic to $\mathbf{Z}\oplus\mathbf{Z}$. There are then two cases, one when the abelian subgroup is cyclic generated by a loxodromic element, and one when it is contained in a $\mathbf{Z}\oplus\mathbf{Z}$ parabolic subgroup. For the first case, methods of algebraic number theory are used to show that if $\beta, \beta_1, \dots, \beta_j$ are elements of a number field, and β is not a root of unity, then for any positive integer m there is a ring homomorphism from $\mathbf{Z}[\beta, \beta_1, \dots, \beta_j]$ to a finite field F, such that the multiplicative order of the image of β is divisible by m. This implies that for any positive integer m and any $\gamma \in \Gamma$, there exists a homomorphism from Γ to a finite group for which the order of the image of γ is divisible by m, and the first case can be deduced from this. For the second case, the argument uses the representation variety of Γ and the fact that finitely generated subgroups of $\mathrm{PSL}(2, \mathbf{C})$ are residually finite.

D.McCullough (Norman)

AMS subject classification: 20E26;57M07;57M50;57M05;20E07;30F40;20H10
Keywords: separable subgroups; residually finite groups; locally extended residually finite groups; LERF groups; Abelian subgroups; subgroups of finite index; fundamental groups; hyperbolic 3-manifolds; 3-orbifolds; representation varieties; Kleinian groups; number fields