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Residual properties of free groups and probabilistic methods. (English)
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This paper is an interesting application of probabilistic ideas to problems of combinatorial group theory. The main tool is the following Theorem 5. Let S be a finite simple group and let $w(X, Y)$ be a nontrivial element of the free group F_2 on X, Y. Then the probability that two randomly chosen elements $x, y \in S$ satisfy $w(x, y) \neq 1$ tends to 1 as $|S| \to \infty$.

W. Magnus [Noneucledean tesselations and their groups, Academic Press (1974; Zbl 0293.50002)] raised the classical problem: whether or not the free group F_k (with $k > 1$) is residually $\mathcal{J}$ for any infinite set $\mathcal{J}$ of nonabelian finite simple groups. The affirmative answer to this was given by *T. S. Weigel* [in J. Algebra 160, No. 1, 16-41 (1993; Zbl 0805.20024), Commun. Algebra 20, No. 5, 1395-1425 (1992; Zbl 0751.20025), Isr. J. Math. 77, No. 1-2, 65-81 (1992; 0815.20019)]. This result follows from a more general result of the paper under review: Theorem 3. Let S be a finite simple group and let w be a nontrivial element of the free group $F_2 = \langle X, Y \rangle$. Then the probability that two randomly chosen elements $x, y \in S$ satisfy both $\langle x, y \rangle = S$ and $w(x, y) \neq 1$ tends to 1 as $|S| \to \infty$.

Another application of the main argument is the following analogue of the well known Tits alternative: Theorem 8. Let Γ be a finitely generated group which is linear over a field K, and G its profinite completion. Then either Γ is virtually solvable, or G has an open subgroup G_0 having a dense free subgroup F of finite rank.

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