Zentralblatt–MATH

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Residual nilpotency of Fuchsian groups. (English)

Ill. J. Math. 28, 299-311 (1984).

A cocompact Fuchsian group Γ has a signature $(g; m_1, m_2, ..., m_r)$ which indicates, in particular, that Γ has r conjugacy classes of maximal finite cyclic subgroups and their orders are $m_1, m_2, ..., m_r$. All these groups are residually finite and except for the cases when $g = 0$ and the m_i are coprime in pairs, in which case Γ is perfect, are known to be residually finite-and-soluble [*C. H. Sah*, Acta. Math. 123, 13-42 (1969; Zbl. 208, 100)]. The author proves that these Fuchsian groups are residually nilpotent if and only if the signature is p-local i.e. the m_i are all powers of a single prime (this includes the torsion-free case). More generally, the study of homomorphisms from cocompact Fuchsian groups to nilpotent groups is reduced to the study of homomorphisms from p-local groups to finite p-groups. Using this, homomorphisms (with torsion free kernel) onto nilpotent groups are characterized. These are of particular interest as they give rise to nilpotent automorphism groups of Riemann surfaces.

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