Zentralblatt–MATH

Menth, Martin

Nilpotent groups with every quotient residually finite. (English)

J. Group Theory 5, No.2, 199-217 (2002). [ISSN 1433-5883; ISSN 1435-4446]
http://www.deGruyter.de/journals/jgt

The residually finite groups, all factor-groups of which are residually finite ($(\mathbf{RF})^{\mathbf{Q}}$-groups) are investigated. Important examples of such groups one can find among soluble groups with finiteness conditions: by P. Hall's classical theorem every finitely generated Abelian-by-nilpotent group is residually finite, i.e. every such group is an $(\mathbf{RF})^{\mathbf{Q}}$-group. FC-groups provide us with other examples of $(\mathbf{RF})^{\mathbf{Q}}$-groups. In fact, every periodic FC-group with finite Sylow p-subgroups for each prime p and every periodic locally soluble FC-group with Abelian Sylow p-subgroups for each prime p is an $(\mathbf{RF})^{\mathbf{Q}}$-group.

The author describes the structure of nilpotent $(\mathbf{RF})^{\mathbf{Q}}$-groups. Let G be an Abelian torsion-free group of finite 0-rank, B a finitely generated subgroup such that G/B is periodic. Put $S_p(G) = \{p \mid \text{a Sylow } p\text{-subgroup of } G/B \text{ is infinite}\}$. If G is a nilpotent torsion-free group of finite 0-rank and $\langle 1\rangle = Z_0 \leq Z_1 \leq \cdots \leq Z_n = G$ is the upper central series of G, then put $S_p(G) = \bigcup_{0\leq j\leq n-1} S_p(Z_{j+1}/Z_j)$.

Theorem 5.15. Let G be a nilpotent group and T the periodic part of G. Then G is an $(\mathbf{RF})^{\mathbf{Q}}$-group if and only if the following condition holds: (1) The Sylow p-subgroup of G is Abelian-by-finite and bounded for each prime p; (2) G/T has finite 0-rank and $S_p(G/T) = \emptyset$.

Note that some proofs are unnecessary complicated. For instance, proving Lemma 7.1 the author uses Lemma 3.2, which is based on very deep results of the theory of residually finite groups. However directly from the definition it follows that a subdirect product of any, not necessarily finite, set of quasicyclic groups has a quasicyclic factor-group and hence is not residually finite.

Igor Subbotin (Los Angeles)

AMS subject classification: 20E26;20F18
Keywords: residually finite groups; factor-groups; periodic FC-groups; nilpotent torsion-free groups; upper central series