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Raptis, E.; Talelli, O.; Varsos, D.

On the Hopficity of certain HNN-extensions with base a Baumslag-Solitar group. (English)

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The authors study the class of groups

$$G = \langle t, a, b \mid ta^{\nu}t^{-1} = b^{\xi},\ ba^{\lambda}b^{-1} = a^{\mu}\rangle$$

with respect to Hopficity. The authors give a characterization of the residual finiteness, the Hopficity, and the automorphism group of G in terms of the integers ν, ξ, λ, μ.

It is shown that G is residually finite if and only if $|\lambda| = |\mu|$ (Theorem 2.6). The group G is an HNN-extension with base a Baumslag-Solitar group $K = \langle a, b \mid ba^{\lambda}b^{-1} = a^{\mu}\rangle$, hence, it admits an action without inversions on a tree T_G, the standard tree associated to the presentation $G = \langle t, K \mid ta^{\nu}t^{-1} = b^{\xi}\rangle$ [see *W. Dicks, M. J. Dunwoody*, Groups acting on graphs, Cambridge Studies in Advanced Mathematics, 17. Cambridge etc.: Cambridge University Press (1989; Zbl 0665.20001) and *J.-P. Serre*, Trees, Transl. from the French by John Stillwell. Berlin-Heidelberg-New York: Springer-Verlag (1980; Zbl 0548.20018)].

The authors prove that if $|\lambda| \neq |\mu|$ and every epimorphism of G sends K to itself up to conjugation, then G is non-Hopfian if and only if either $|\lambda| \neq 1$, $\lambda \mid \nu$, $\lambda \mid \mu$, $(\mu/\lambda, \lambda) = 1$, or $|\mu| \neq 1$, $\mu \mid \nu$, $\mu \mid \lambda$, $(\lambda/\mu, \mu) = 1$ (Theorem 4.4). It is proved that every epimorphism of G sends K to itself up to conjugation when $\xi = 1$ (Proposition 4.5). Hence if $\xi = 1$ and $|\lambda| \neq |\mu|$ then G is non-Hopfian if and only if either $|\lambda| \neq 1$, $\lambda \mid \nu$, $\lambda \mid \mu$, $(\mu/\lambda, \lambda) = 1$, or $|\mu| \neq 1$, $\mu \mid \nu$, $\mu \mid \lambda$, $(\lambda/\mu, \mu) = 1$ (Theorem 4.6). Moreover, in this case, the authors show that essentially $\mathrm{Aut}(G)$ splits as an HNN-extension, by exhibiting an action without inversions, of $\mathrm{Aut}(G)$ on T_G with quotient a loop (Theorem 5.2). The arguments use mostly the tree aspect of HNN-extensions.

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