Zentralblatt–MATH

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Conjugacy separability and free products of groups with cyclic amalgamation. (English)

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A group G is conjugacy separable if for any pair of non-conjugate elements x, y in G there is a finite image $\overline{G}$ of G in which the images $\overline{x}$ and $\overline{y}$ are non-conjugate. It is well known that finitely generated conjugacy separable groups have solvable conjugacy problems. This paper considerably enlarges the class of groups known to be conjugacy separable.

A class $\mathcal{X}$ of groups is defined by five (natural but slightly technical) conditions relating to the profinite topology, the first condition being that of conjugacy separability. The main results proved are: Theorem A. Let G_1 and G_2 be in $\mathcal{X}$. Then their free product $G_1 *_H G_2$ amalgamating a cyclic common subgroup H is again in $\mathcal{X}$ (and hence in particular is conjugacy separable). Theorem B. The class $\mathcal{X}$ contains all polycyclic-by-finite groups, all free-by-finite groups, all Fuchsian groups, and all surface groups.

It follows that any group obtained from polycyclic-by-finite groups and free-by-finite groups by repeatedly forming free products with cyclic amalgamation is conjugacy separable. This generalizes results of *B. Fine* and *G. Rosenberger* [Contemp. Math. 109, 11-18 (1990; Zbl 743.20045)], *C. Y. Tang* [Can. Math. Bull. 38, 120-127 (1995; Zbl 822.20027), J. Pure Appl. Algebra 120, No. 2, 187-194 (1997; Zbl 881.20012)] and *L. Ribes* and *P. A. Zalesskii* [J. Algebra 179, No. 3, 751-774 (1996; Zbl 869.20015)].

The proof of Theorem A depends on analysing the action of a profinite amalgamated free product on the associated profinite tree, developing methods introduced by Ribes and Zalesskii [loc. cit.]. The main new ingredient in Theorem B is the claim regarding polycyclic-by-finite groups; this is proved by establishing various properties of the profinite completions of such groups, which may be of independent interest.

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