Zentralblatt–MATH

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Faithful linear representations and residual finiteness of some products of cyclics with one relation. (English. Russian original)

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A group of the form $G = \langle a_1, \ldots, a_n \mid a_1^{e_1} = \cdots = a_n^{e_n} = R^m(a_1, \ldots, a_n) = 1\rangle$, where $n \geq 2$, $m \geq 2$, $e_i = 0$ or $e_i \geq 2$ for all i, and $R(a_1, \ldots, a_n)$ is a cyclically reduced word in the free generators $a_1, \ldots, a_n$, properly containing them all, is called a product of cyclics with one relation. In [Proc. Am. Math. Soc. 102, 249-254 (1988; Zbl. 653.20029)] *B. Fine, J. Howie* and the author proved that the Freiheitssatz holds for G; i.e., the subgroup generated by the elements $a_1, \ldots, a_{n-1}$ is a free product of cyclics. The proof uses a representation of G in $\mathrm{PSL}(2, \mathbf{C})$. If this representation is faithful, then G is residually finite by Mal'tsev's theorem. In general, a faithful representation $\rho : G \to \mathrm{PSL}(2, \mathbf{C})$ does not exist. It would be interesting, in this connection, to find conditions on the group G that would guarantee its faithful representability in $\mathrm{PSL}(2, \mathbf{C})$. In the present article, we prove the following partial result. Theorem. Let $G = \langle a_1, \ldots, a_n \mid a_1^{e_1} = \cdots = a_n^{e_n} = (U(a_1, \ldots, a_p) \cdot V(a_{p+1}, \ldots, a_n))^m = 1\rangle$, where $m \geq 2$, $n \geq 3$, $1 \leq p \leq n-2$, $e_i = 0$ or $e_i \geq 2$ for $i = 1, \ldots, n$, $U(a_1, \ldots, a_p)$ is a nontrivial word in the free generators $a_1, \ldots, a_p$, and $V(a_{p+1}, \ldots, a_n)$ of finite order, and not a proper power. Then G admits a faithful representation in $\mathrm{PSL}(2, \mathbf{C})$, and, hence, is residually finite. It follows from this theorem that the fundamental groups of certain 3-manifolds admit faithful representations in $\mathrm{PSL}(2, \mathbf{C})$.

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