Zentralblatt–MATH

Witbooi, Peter

Finite images of groups. (English)

Quaest. Math. 23, No.3, 279-285 (2000). ISSN1607-3606
http://www.nisc.co.za/Journals/Maths.htm

Let $G_k = \langle a, b; a^{-1}ba = b^k\rangle$ and $G_k(m) = \langle a, b;\ a^{-1}ba = b^k,\ b^m = 1\rangle$ be groups defined for pairs of relatively prime positive integers k and m. Let $\mathcal{F}(G)$ be the set of all isomorphism classes of finite quotient groups of the group G and for a set of primes π, let $\mathcal{F}_\pi(G)$ be the set of all groups in $\mathcal{F}(G)$ which are π-groups. Define $\mathcal{C}^{(n)}$ to be the class of all groups G which are semidirect products $G = T \times_w \mathbf{Z}^n$, where T is a finite additive Abelian group and $w\colon \mathbf{Z}^n \to \mathrm{Aut}T$ is such that for each $x \in \mathbf{Z}^n$, there is an integer u_x with $w(x)\colon t \to u_x t$. Put $\mathcal{C} = \bigcup_n \mathcal{C}^{(n)}$.

The author establishes the following three main results: (1) Let M, N be finitely generated groups with finite commutator subgroups and K a group in $\mathcal{C}$ with torsion subgroup of odd order and torsion-free quotient which is non-cyclic. If K is either nilpotent or the exponent m of its torsion subgroup is a prime power, then $\mathcal{F}(N \times K) = \mathcal{F}(M \times K)$ if and only if $N \times K \simeq M \times K$. (2) A necessary and sufficient condition for $\mathcal{F}_\pi(G_k(m)) = \mathcal{F}_\pi(G_k(n))$, where m and n are related to π in a certain manner, is obtained. (3) If k and ℓ are such that $\mathcal{F}(G_k(m)) = \mathcal{F}(G_\ell(m))$ for every positive m which is relatively prime to both k and ℓ then $k = \ell$.

C.G.Chehata (Alexandria)

AMS subject classification: 20F05;20E26;20E22